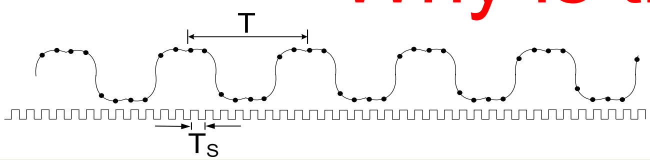
EE 435

Lecture 30

Data Converter Characterization

- Quantization Noise
- Absolute Accuracy
- Relative Accuracy

Why is this a Key Theorem?



$$x(t) = A_0 + \sum_{k=1}^{h-1} A_k \sin(k\omega t + \theta_k)$$

Review from last lecture.

THEOREM: Consider a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$|A_m| = \frac{2}{N}|X(mN_P+1)| \qquad 0 \le m \le h-1$$

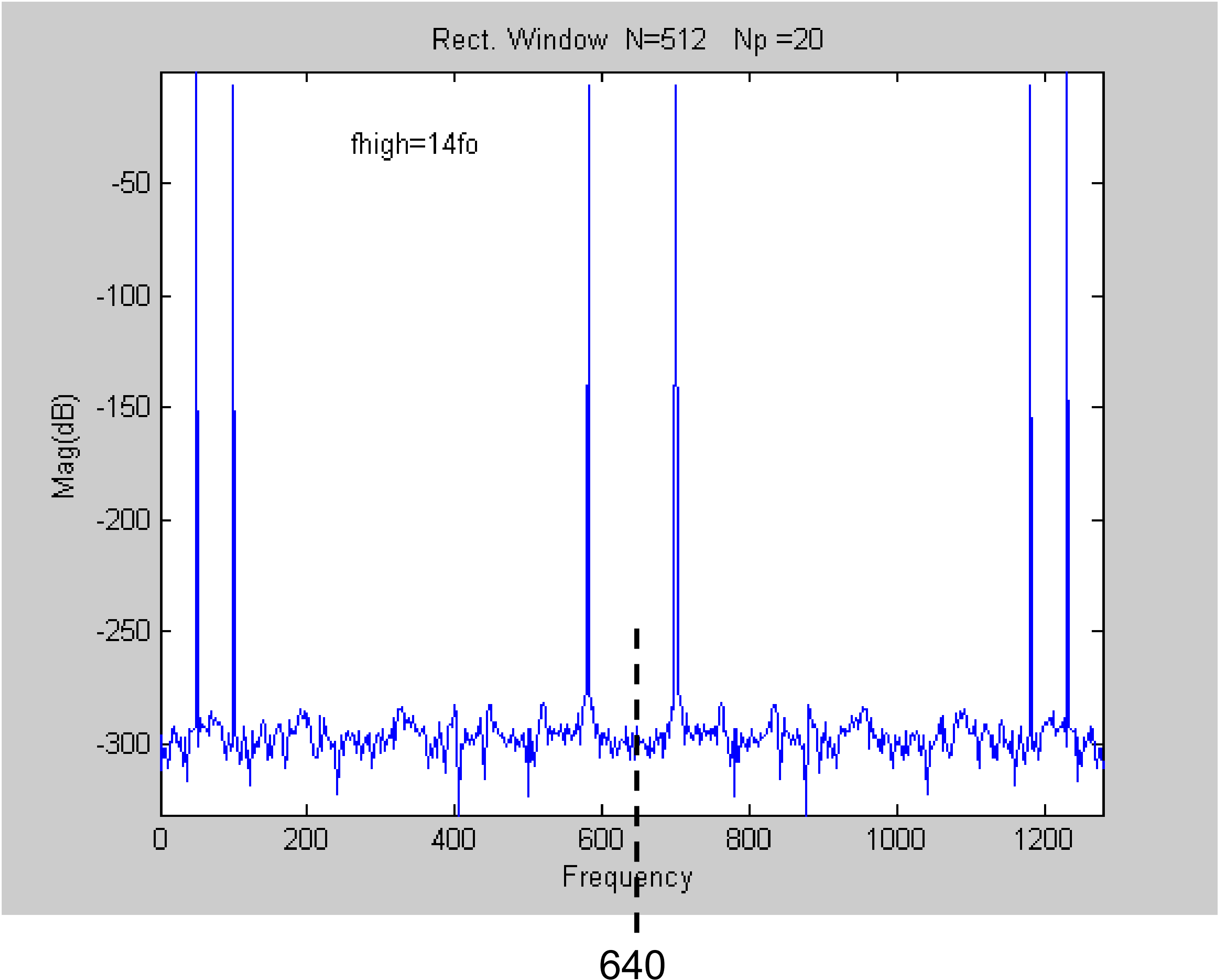
and $X(k)=0$ for all k not defined above

where $\langle X(k)\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_S)\rangle_{k=0}^{N-1}$

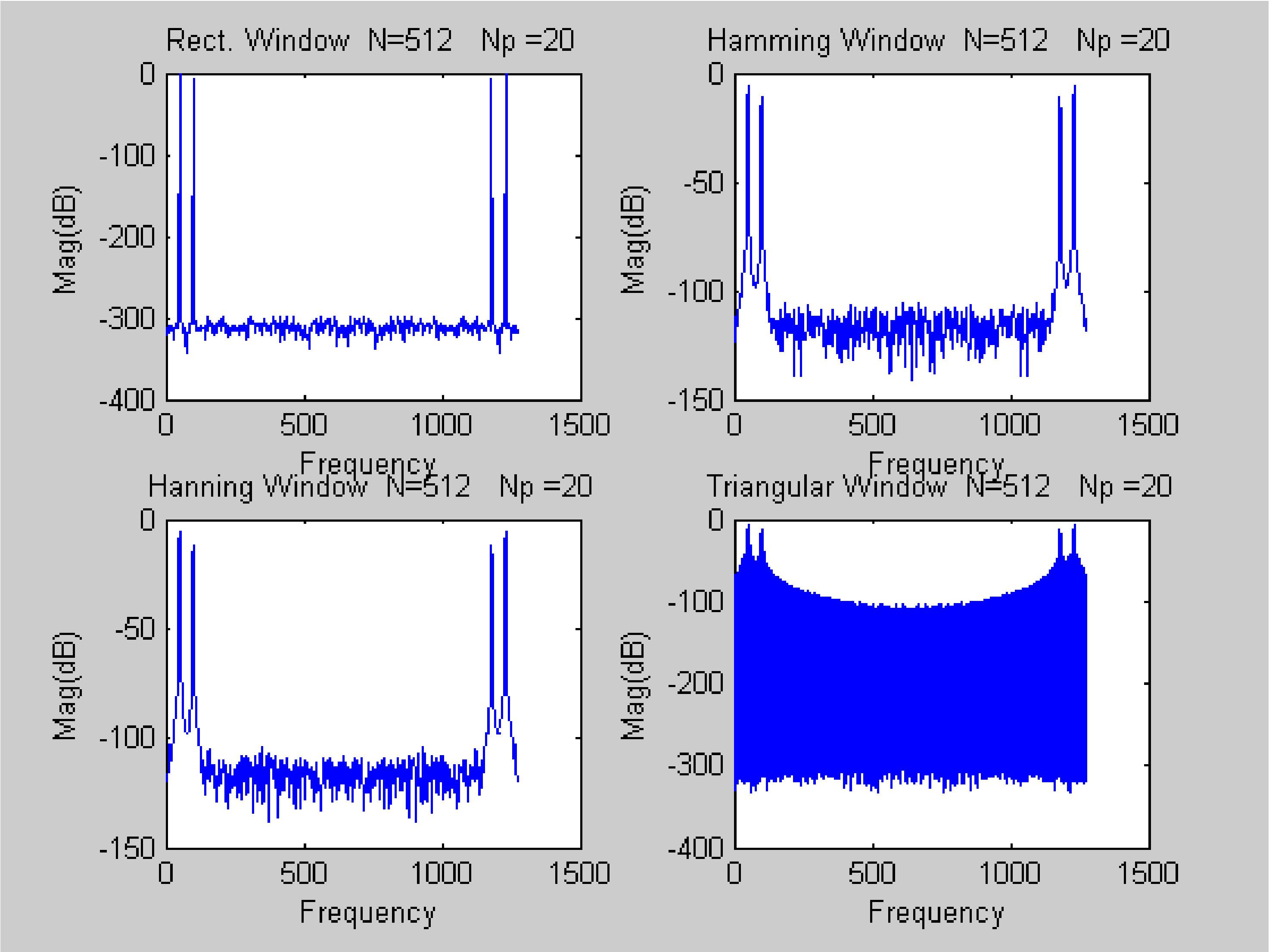
$\langle A_k \rangle$ are the Fourier Series Coefficients, N=number of samples, N_P is the number of periods, and $h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem
- If "signal" is output of a system (e.g. ADC or DAC), f_{MAX} is independent of f

Effects of High-Frequency Spectral Components



Comparison of 4 windows when sampling hypothesis are satisfied



Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Spectral Characterization of Data Converters

- Distortion Analysis
- Time Quantization Effects
  - of DACs
  - of ADCs
- Amplitude Quantization Effects
  - of DACs
  - of ADCs

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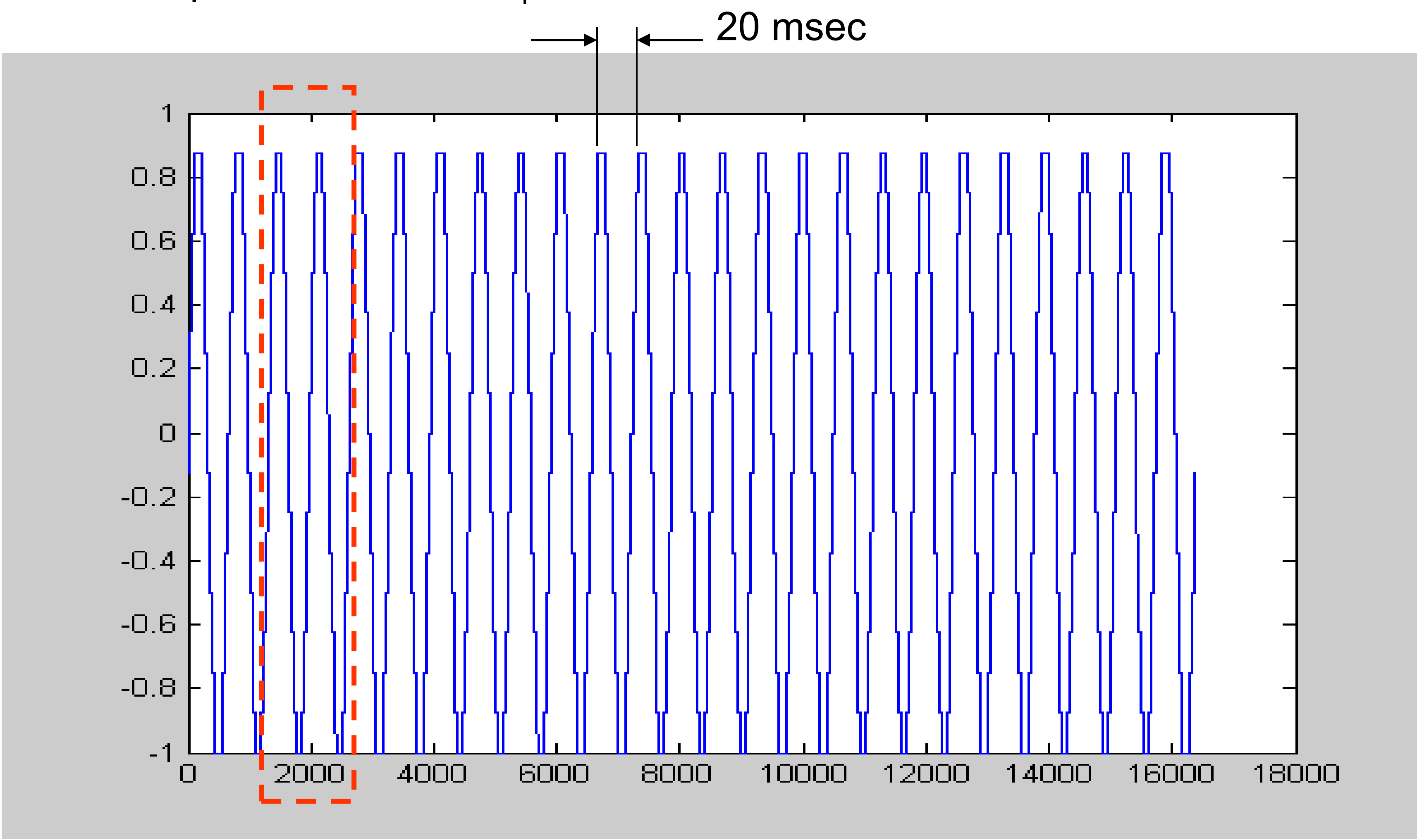
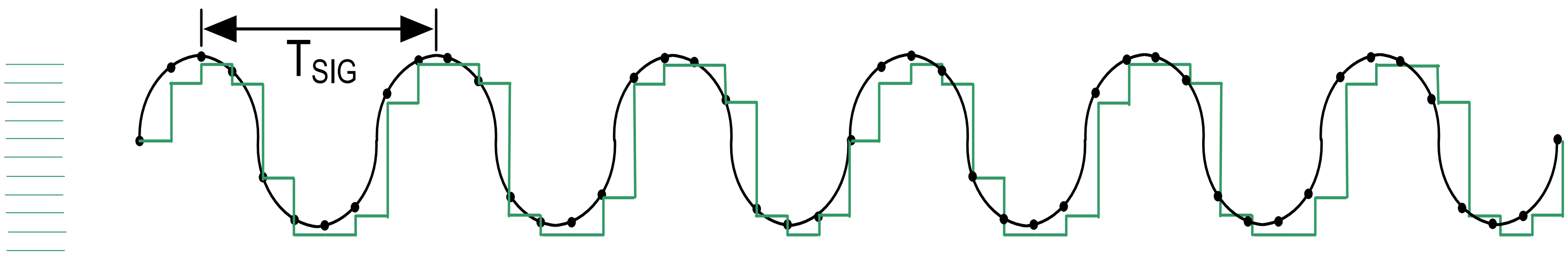
Review from last lecture

Quantization Effects

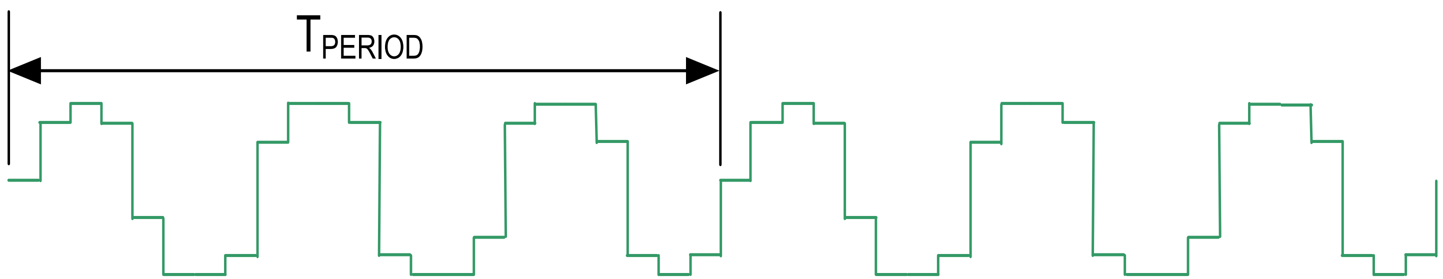
16,384 pts   res = 4bits N_P=25



Spectral Characteristics of DAC

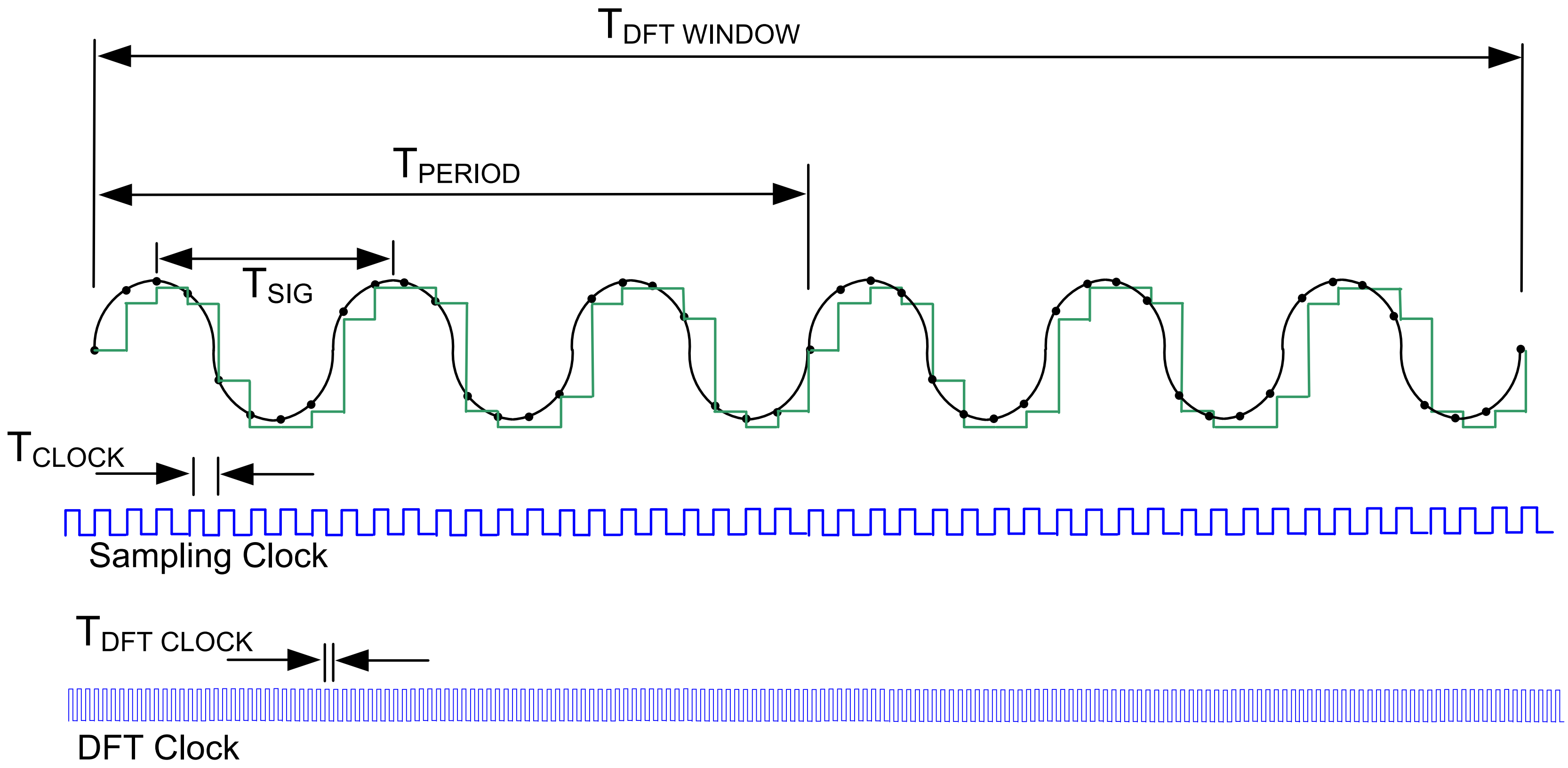


Quantization Levels

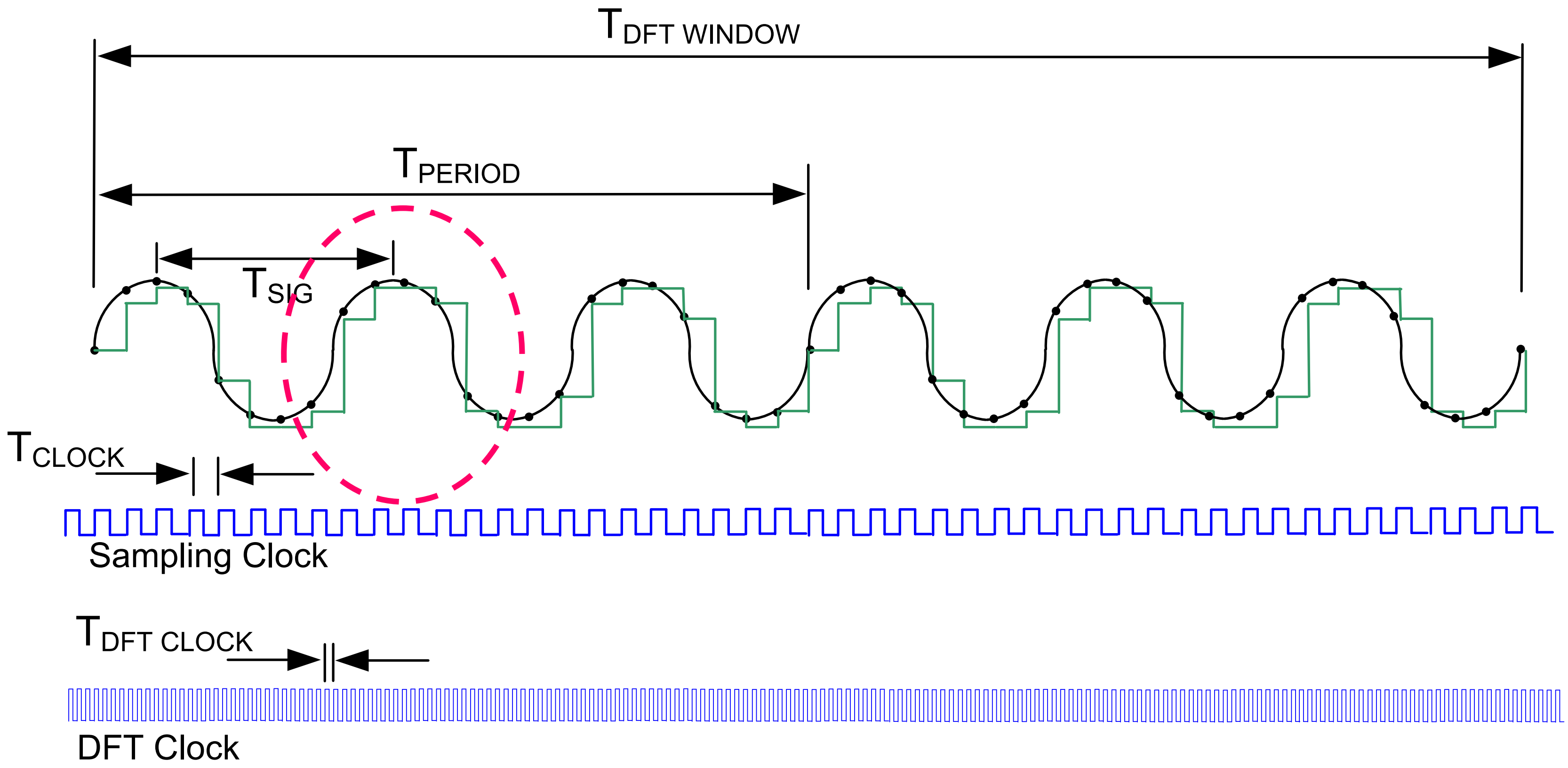


Quantized Sampled Input Signal (with zero-order sample and hold)

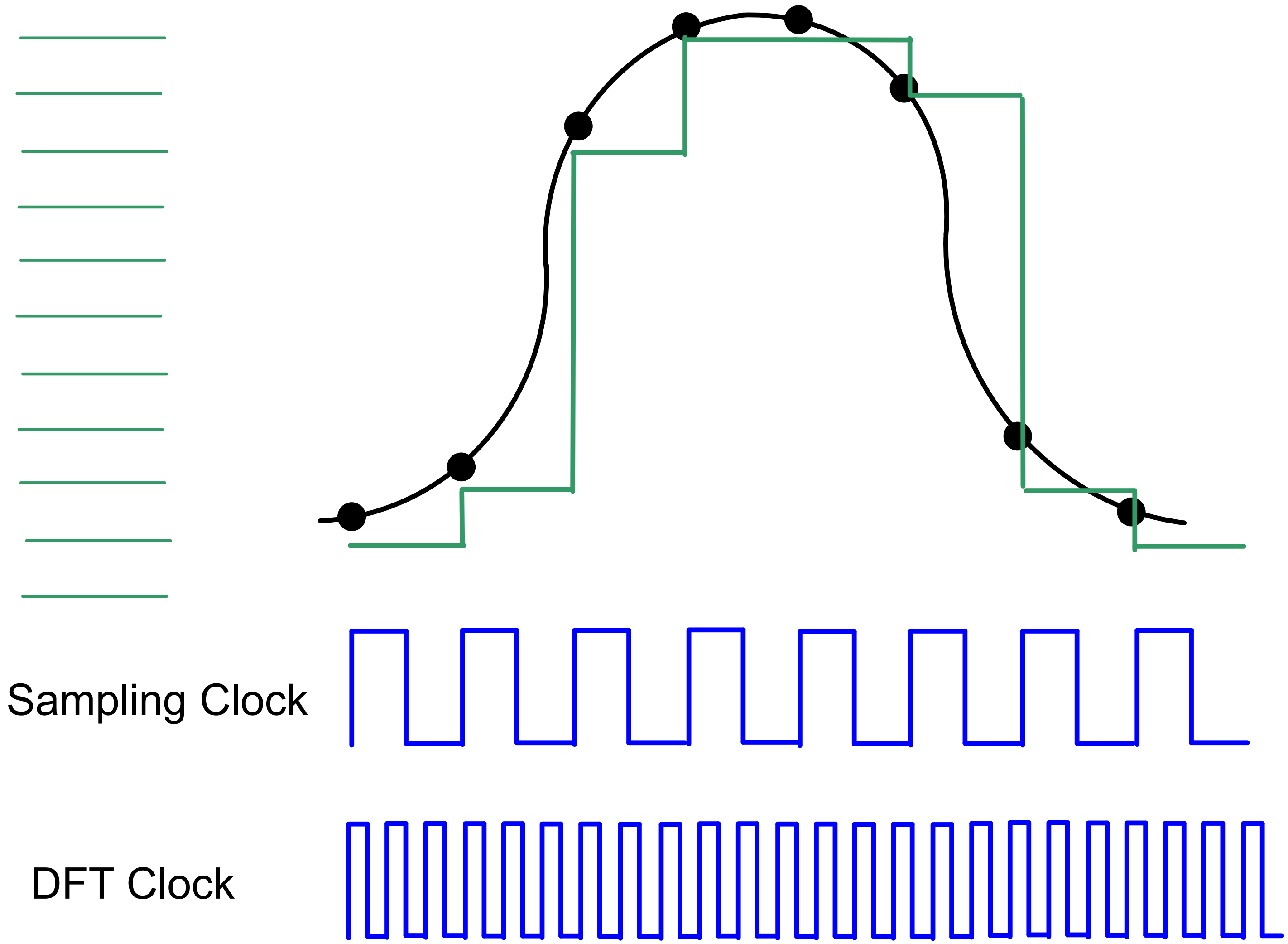
Spectral Characteristics of DAC



Spectral Characteristics of DAC

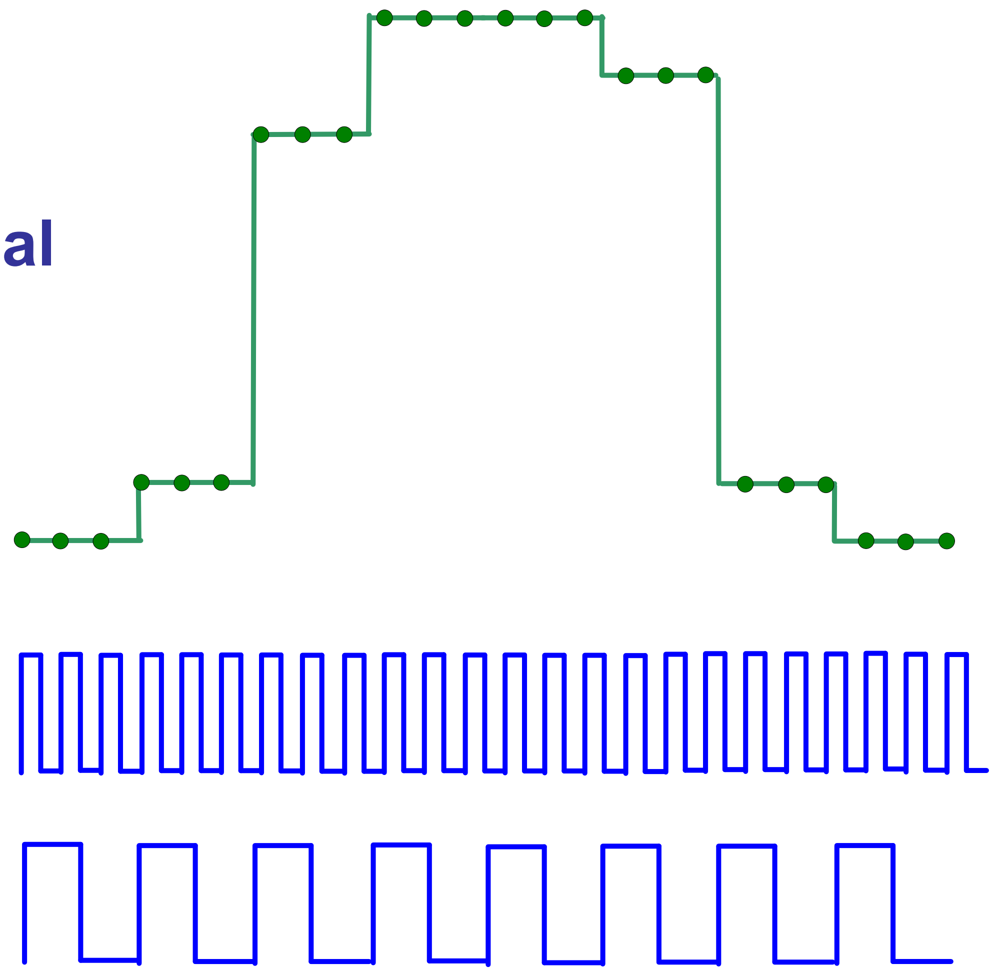


Spectral Characteristics of DAC



Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)

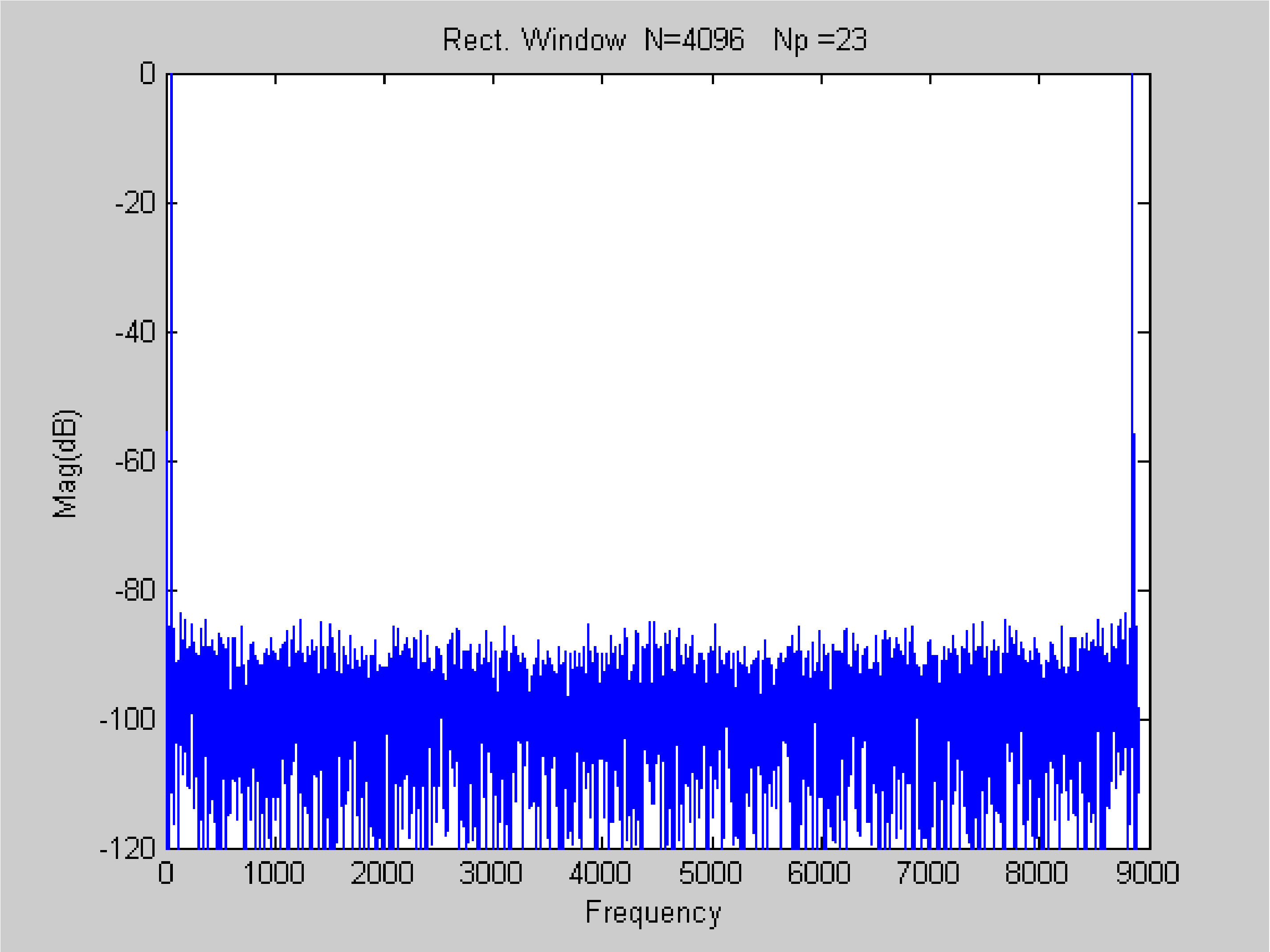


DFT Clock

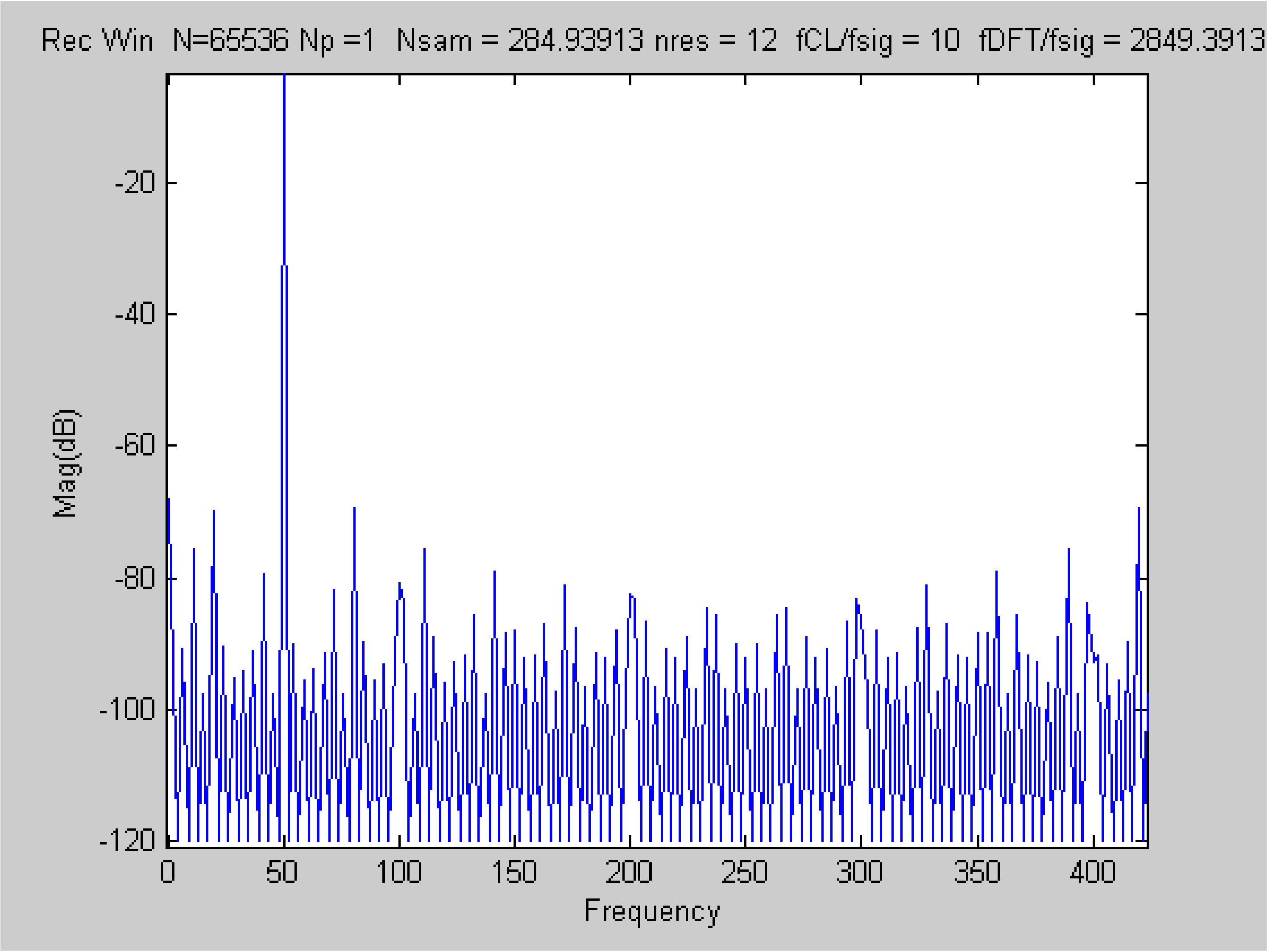
Sampling Clock

Quantization Effects

Res = 10 bits



DFT Simulation from Matlab



Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce harmonic distortion

Time and amplitude quantization do increase the noise floor

Jitter

THEOREM: Consider a periodic signal with period $T=1/f$ and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$\left|A_m\right| = \frac{2}{N}\left|X(mN_P+1)\right| \qquad 0 \le m \le h-1$$

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where $\langle X(k)\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_S)\rangle_{k=0}^{N-1}$

$\langle A_k \rangle$ are the Fourier Series Coefficients, N=number of samples, N_P is the number of periods, and $h = \mathrm{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

Another part of the hypothesis: Samples should be uniformly spaced !

If samples are not uniformly spaced, the deviation from uniformly spacing is termed **jitter**

Jitter

Jitter often comprised of two parts:

1. Random variations in clock edges due to random movement of electrons

2. Circuit structures used to generate sampling clocks (part of this can be termed clock skew)

Jitter also causes the DFT to not correctly represent the Fourier Series Coefficients

Effects Can be Significant

Can introduce distortion, harmonics, spectral tones that are not harmonics, and increase the noise floor

A concern for both characterization (testing) and design of data converters

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Quantization Noise

- DACs and ADCs generally quantize both amplitude and time
- If converting a continuous-time signal (ADC) or generating a desired continuous-time signal (DAC) these quantization's cause a difference in time and amplitude from the desired signal – this difference is termed “noise”.
- First a few comments about Noise

What is Noise in a data converter?

Noise is a term applied to some nonideal effects of a data converter

Precise definition of noise is probably not useful

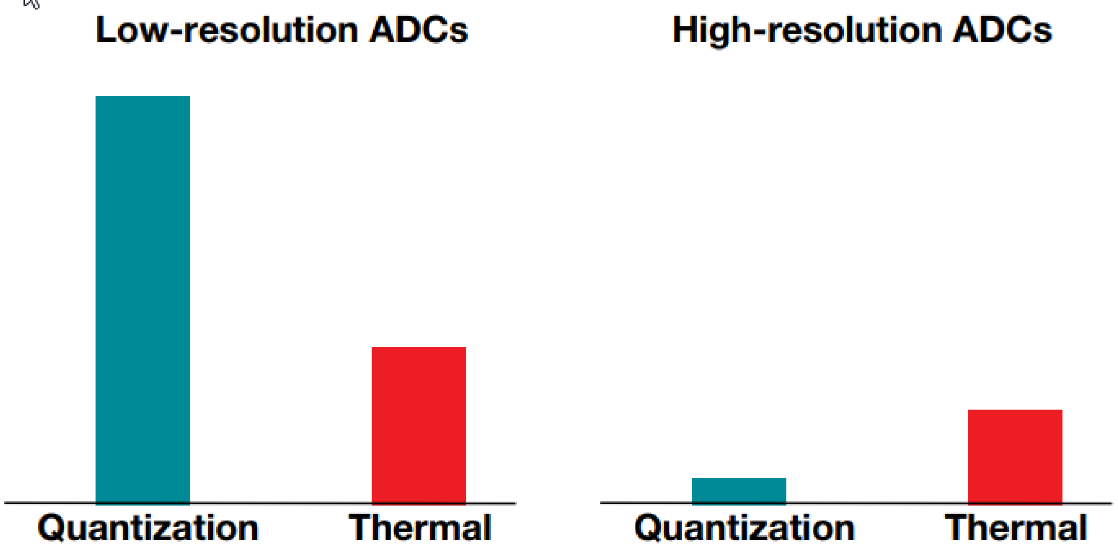
Some differences in views about what nonideal characteristics of a data converter should be referred to as noise

Types of noise:

- Random noise due to movement of electrons in electronic circuits (resistors and active devices) – highly dependent upon temperature thus often termed “thermal” noise
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
  - Quantization noise
  - Sample Jitter
  - Harmonic Distortion

Noise

Noise is any undesired signal (typically random) that adds to the desired signal, causing it to deviate from its original value.[1]



[1] from Bryan Lizon

[1]Definitiion from "Fundamentals of Precision ADC Noise Analysis", Bryan Lizon, Texas Instruments, Sept. 2020

Note: This is conceptual but not precise since "desired" signal will depend, in part, upon what a user desires. There are also undesired bandwidth limitations on most data converters that would create very large "noise" for higher-frequency inputs based upon this definition.

Noise

Good reference on noise in ADCs

Fundamentals of Precision ADC Noise Analysis

Design tips and tricks to reduce noise with delta-sigma ADCs

Texas Instruments

ti.com/precisionADC September | 2020 by Bryan Lizon

file:///C:/Users/rlgeiger/Documents/ABIN/Research/Noise/slyy192.pdf

Noise

Some major types of noise:

- Random noise due to movement of electrons in electronic circuits (thermal noise)
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits

All of these types of noise are present in data converters and are of concern when designing most data converters

Can not eliminate any of these noise types but with careful design can manage their effects to certain levels

Noise (in particular the random noise) is often the major factor limiting the ultimate performance potential of many if not most data converters

Noise

Some major types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits

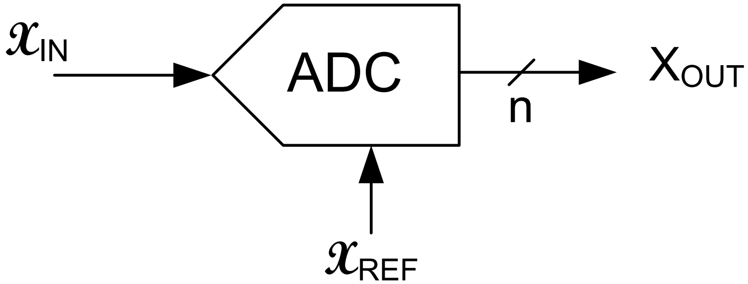
Quantization noise is a significant component of this noise in ADCs and DACs and is present even if the ADC or DAC is ideal

Will now investigate quantization noise

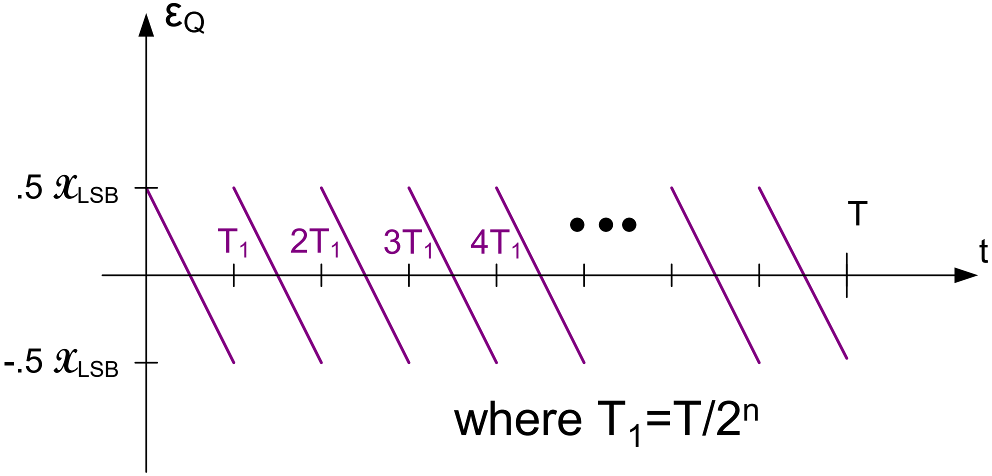
Quantization Noise in ADC

(same concepts apply to DACs)

Consider an Ideal ADC with first transition point at $0.5X_{LSB}$



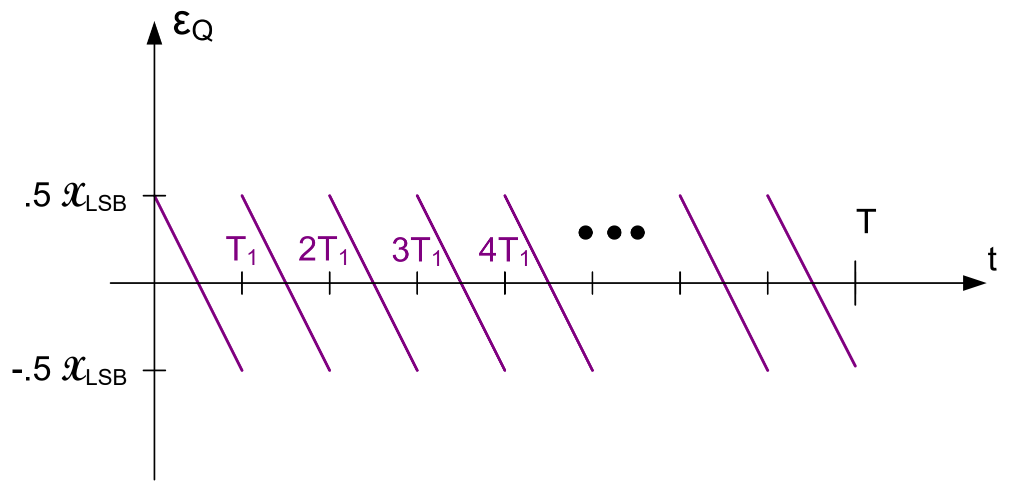
If the input is a low frequency sawtooth waveform of period T that goes from 0 to X_{REF} and if sampling is very fast so that the digital output always represents a quantized version of the input, the error signal in the time domain will be:



where $T_1 = T/2^n$

This time-domain waveform is termed the Quantization Noise for the ADC with a sawtooth (or triangular) input

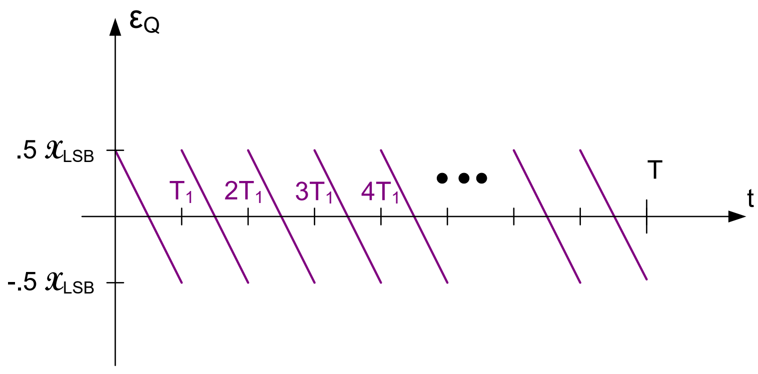
Quantization Noise in ADC

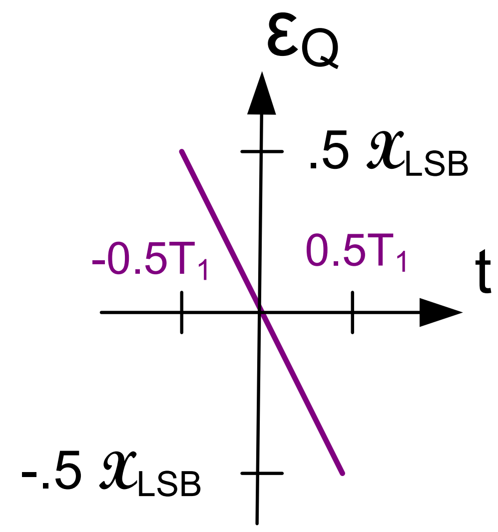


For large n, this periodic waveform behaves much like a random noise source that is uncorrelated with the input and can be characterized by its RMS value which can be obtained by integrating over any interval of length $T_{1.}$ For notational convenience, shift the waveform by $T_1/2$ units

$$E_{RMS} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^{\,2}(t)\,dt}$$

Quantization Noise in ADC





$$E_{RMS} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^{\,2}(t)\, dt}$$

In this interval, ε_Q can be expressed as

$$\varepsilon_Q(t) = -\left(\frac{X_{LSB}}{T_1}\right) t$$

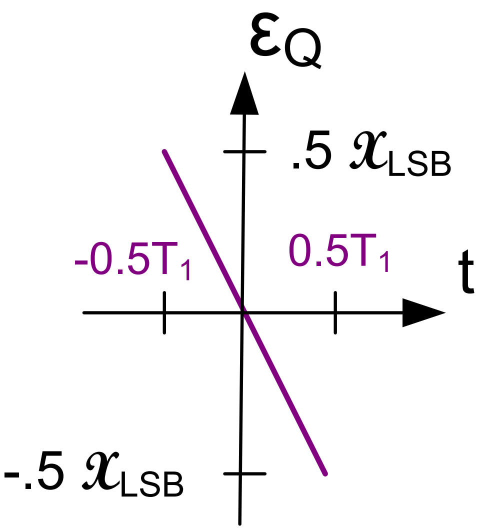
Quantization Noise in ADC

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q^{\,2}(t)\,dt}$$

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \left(-\frac{x_{LSB}}{T_1}\right)^2 t^2\,dt}$$

$$E_{RMS} = x_{LSB}\sqrt{\frac{1}{T_1^3}\left.\frac{t^3}{3}\right|_{-T_1/2}^{T_1/2}}$$

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$



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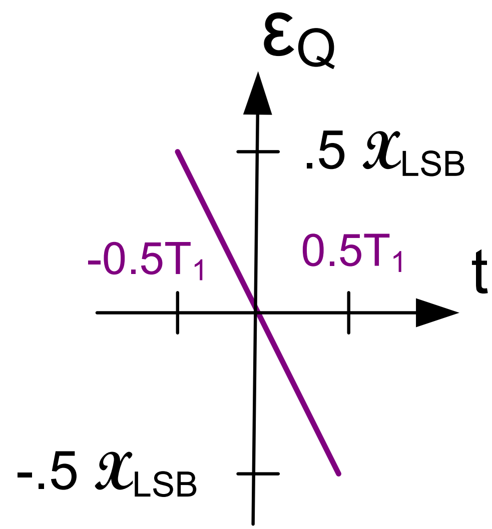
Quantization Noise in ADC

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q^{\,2}(t)\,dt}$$



$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$

$$\varepsilon_Q(t) = -\left(\frac{X_{LSB}}{T_1}\right)t$$

Question: How does this change if the input sawtooth waveform goes from $.25V_{REF}$ to $.75V_{REF}$?

Question: How does this change if the input sawtooth waveform goes from 0 to $.25V_{REF}$?

Quantization noise is almost independent of the signal swing of the input !

Quantization Noise in ADC

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period T and amplitude X_{REF}, it follows by the same analysis that it has an RMS value of

$$x_{RMS} = \frac{x_{REF}}{\sqrt{12}} = \frac{2^n X_{LSB}}{\sqrt{12}}$$

Thus the SNR is given by

$$SNR = \frac{x_{RMS}}{E_{RMS}} = \frac{2^n x_{LSB}}{x_{LSB}} = 2^n$$

or, in dB,

$$SNR_{dB} = 20(n \bullet \log 2) = 6.02n$$

Note: dB subscript often neglected when not concerned about confusion

Quantization Noise in ADC

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period T and amplitude X_{REF}, it follows by the same analysis that it has an RMS value of

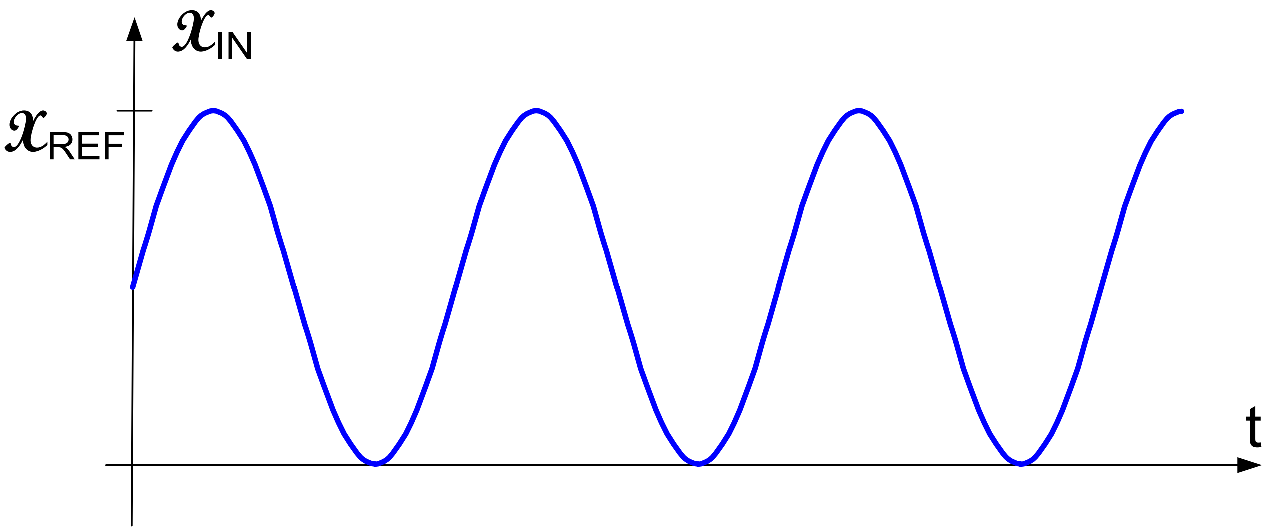
$$SNR_{dB} = 20(n \bullet \log 2) = 6.02n$$

Question: How does the quantization SNR change if the amplitude of sawtooth is only $X_{REF}/2$?

Drops by 6dB!

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?

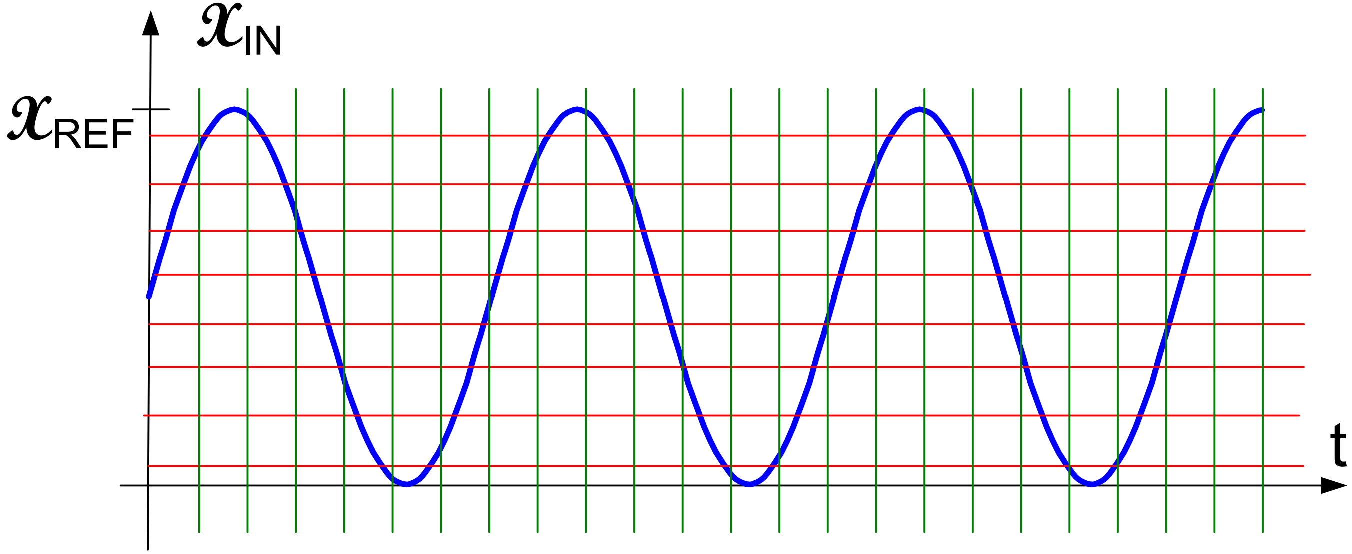


For full-scale sawtooth (or triangular input) $SNR = 20(n \bullet \log 2) = 6.02n$

For full-scale sinusoidal input $SNR = ???$

Quantization Noise in ADC

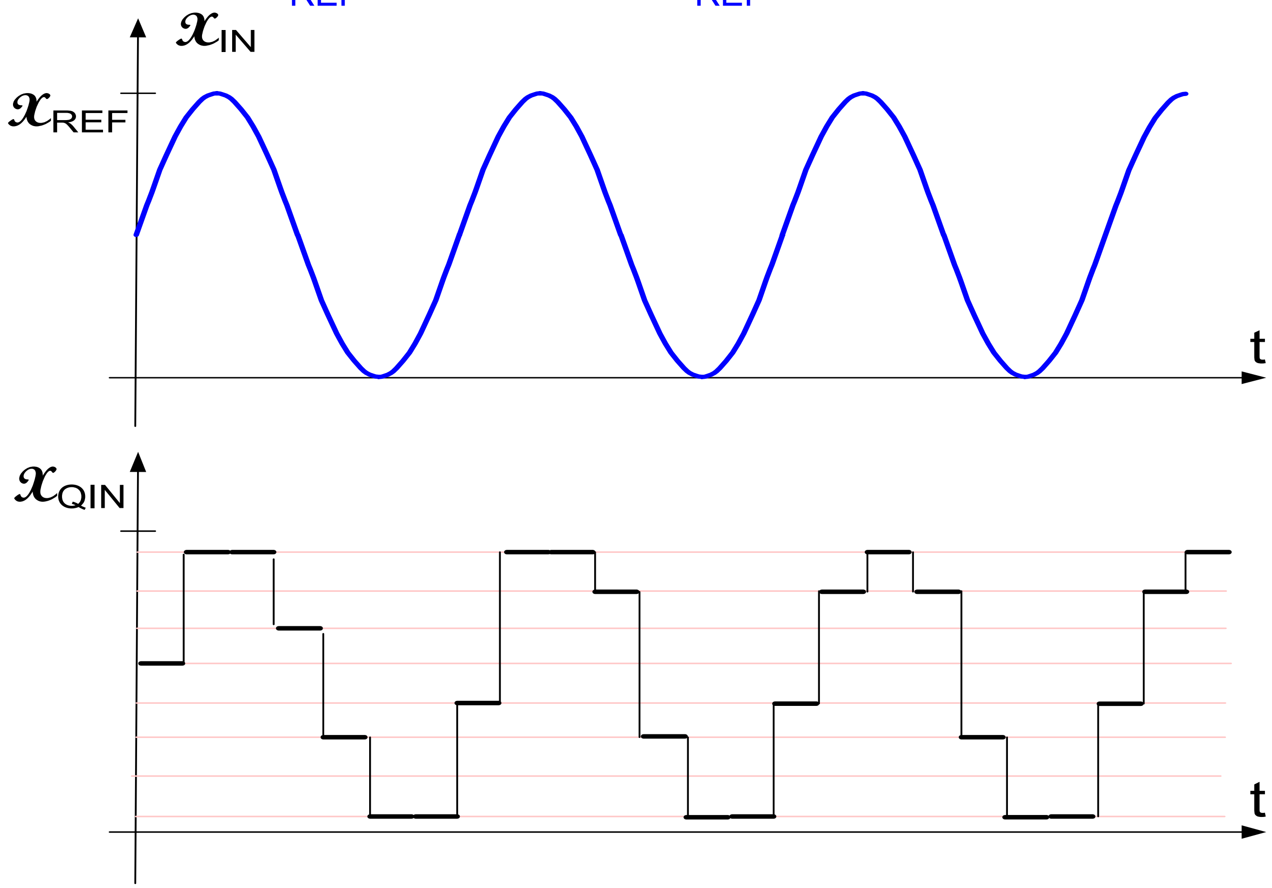
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Time and amplitude quantization points

Quantization Noise in ADC

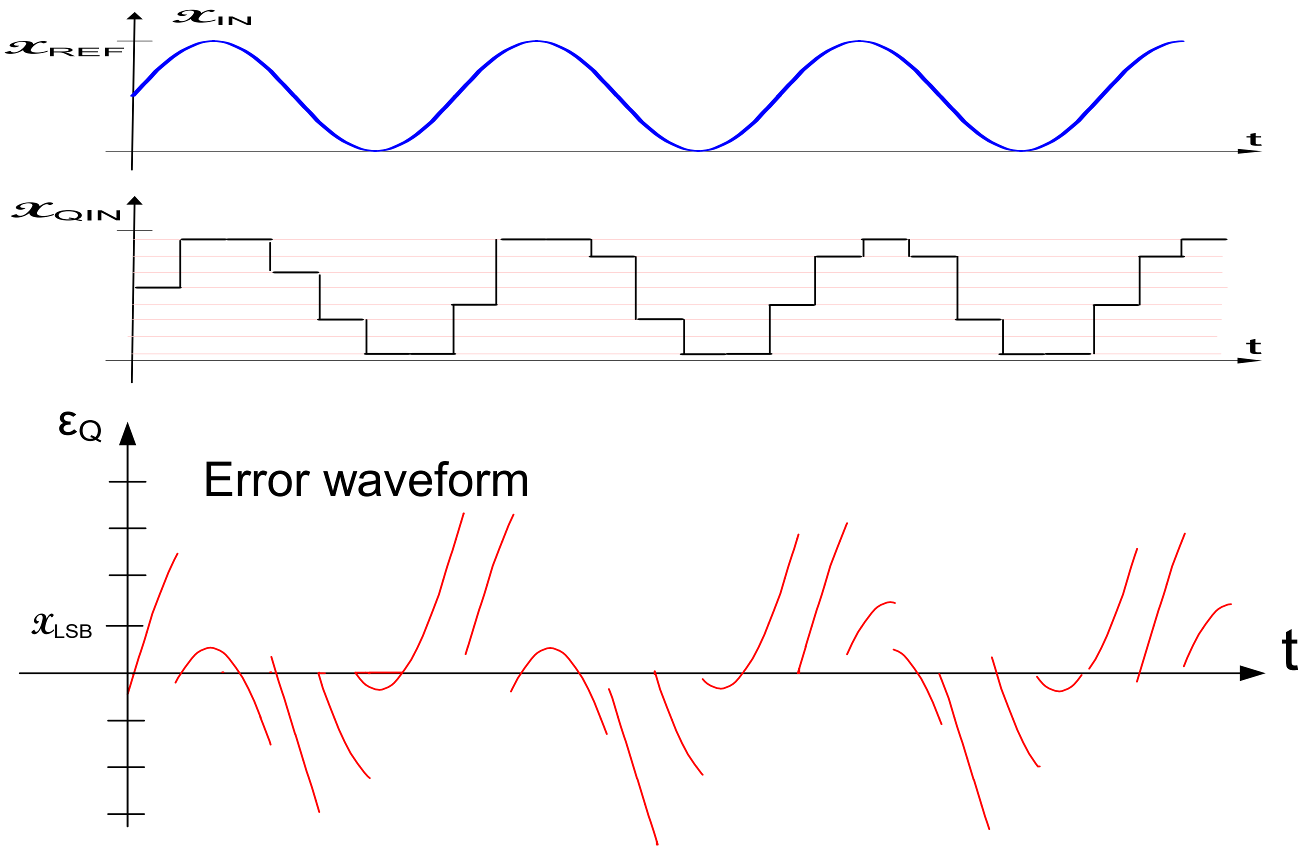
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Time and Amplitude Quantized Waveform

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?

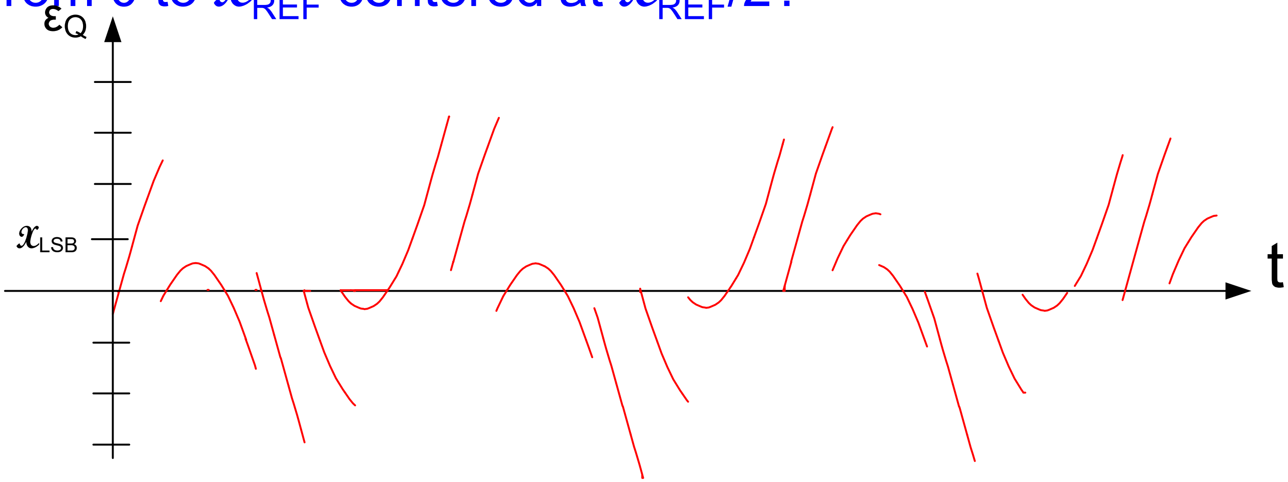


Is the major source of the peak value of this error due to time-quantization or amplitude quantization?

Time Quantization !

Quantization Noise in ADC

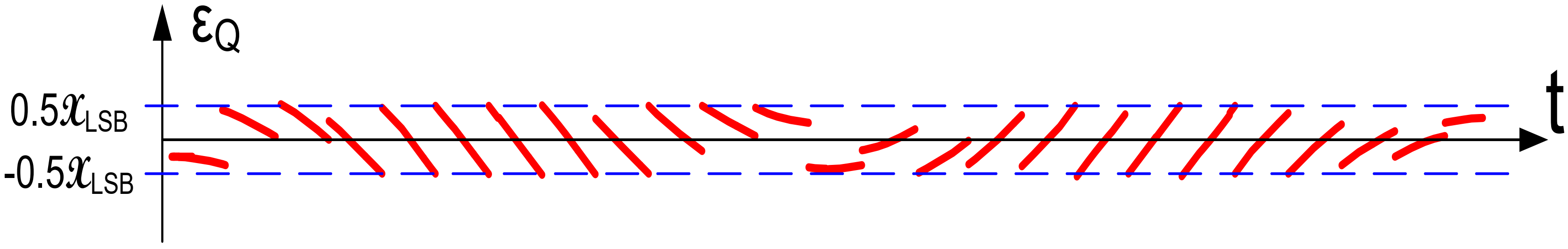
How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



- Appears to be highly uncorrelated with input even though deterministic
- Mathematical expression for ε_Q very messy
- Excursions exceed X_{LSB}
- For low frequency inputs and higher resolution, at any time, errors are approximately uniformly distributed between $-X_{LSB}/2$ and $X_{LSB}/2$
- Analytical form for ε_{QRMS} essentially impossible to obtain from $\varepsilon_Q(t)$

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



For low f_{SIG}/f_{CL} ratios, bounded by $\pm 0.5\ X_{LSB}$ and at any point in time, behaves almost as if a uniformly distributed random variable

$$\varepsilon_Q \sim U[-0.5X_{LSB},\ 0.5X_{LSB}]$$

Quantization Noise in ADC

Recall:

If the random variable f is uniformly distributed in the interval [A,B]
$f : U[A,B]$ then the mean and standard deviation of f are given by

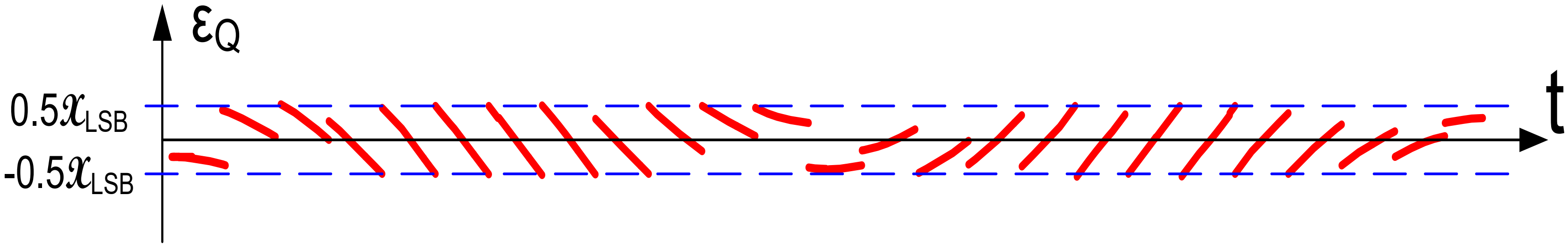
$$\mu_f = \frac{A+B}{2} \qquad \sigma_f = \frac{B-A}{\sqrt{12}}$$

Theorem: If n(t) is a random process and $\langle n(kT_S) \rangle$ is a sequence of samples of n(t) then for large T/T_S,

$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} n^2(t)\,dt} = \sqrt{\sigma^2_{n(kT_S)} + \mu^2_{n(kT_S)}}$$

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



$$\varepsilon_Q \sim U[-0.5X_{LSB},\ 0.5X_{LSB}]$$

$$\mu_{\varepsilon_Q} = \frac{A+B}{2} = 0 \qquad \sigma_f = \frac{B-A}{\sqrt{12}} = \frac{X_{LSB}}{\sqrt{12}}$$

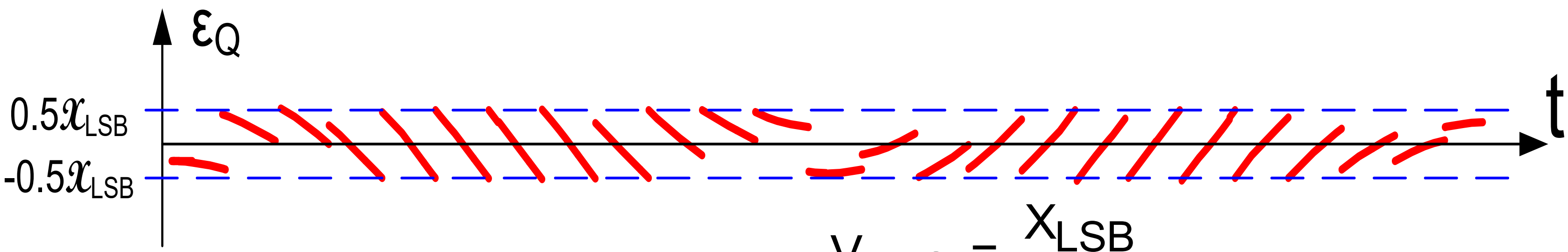
$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} n^2(t)\,dt} = \sqrt{\sigma_n^2 + \mu_n^2}$$

$$V_{RMS} = \sigma_{\varepsilon_Q} = \frac{X_{LSB}}{\sqrt{12}}$$

Note this is the same RMS noise that was present with a triangular input

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



$$V_{RMS} = \frac{X_{LSB}}{\sqrt{12}}$$

But $$V_{INRMS} = \left(\frac{X_{REF}}{2}\right)\frac{1}{\sqrt{2}}$$

Thus obtain

$$SNR = \frac{\dfrac{X_{REF}}{2\sqrt{2}}}{\dfrac{X_{LSB}}{\sqrt{12}}} = 2^n\sqrt{\frac{3}{2}}$$

Finally, in db,

$$SNR_{dB} = 20\log\left(2^n\sqrt{\frac{3}{2}}\right) = 6.02\,n + 1.76$$

ENOB based upon Quantization Noise Reference

Different factors can cause the SNR or SNDR of an ADC to not be ∞

- Quantization effects
- Device noise
- Interference Noise
- Nonlinear distortion
- Signal amplitude
- Jitter
- Computation errors
- …….

It is often useful to consider how an ADC performs from a SNR or SNDR viewpoint relative to how it would perform if only quantization effects (which are unavoidable) for an otherwise ideal ADC are present

An ENOB relative to an otherwise ideal ADC is often used as a metric for assessing SNR or SNDR performance

For example, consider a 14-bit ADC with a full-signal sinusoidal input that has quantization noise of $\frac{X_{LSB}}{\sqrt{12}} = 0.29X_{LSB}$, device noise with an RMS value of $2X_{LSB}$ and interference noise of 5 X_{LSB}. The total noise is then $5.4X_{LSB}$. Thus its SNR is equivalent to that of a much lower resolution ADC that has only quantization noise present. The resolution of that lower resolution ADC would be termed the ENOB relative to a Quantization Noise Only data converter

ENOB based upon Quantization Noise Reference

$$SNR_{dB} = 6.02\,n + 1.76$$

Solving for n, obtain

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02}$$

Note: could have used the SNR_{dB} for a triangle input and would have obtained the expression

$$ENOB = \frac{SNR_{dB}}{6.02}$$

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter

What is ENOB (using standard sinusoidal reference definition) if only quantization noise present with a full-scale sinusoidal input?

ENOB based upon Quantization Noise Reference

$$SNR_{dB} = 6.02\, n + 1.76$$

Solving for n, obtain

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02}$$

Note: could have used the SNR_{dB} for a triangle input and would have obtained the expression

$$ENOB = \frac{SNR_{dB}}{6.02}$$

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter

What is ENOB if only quantization noise present with a full-scale sinusoidal input?

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02} = \frac{6.02n + 1.76 - 1.76}{6.02} = n$$

ENOB based upon Quantization Noise

For very low resolution levels, the assumption that the quantization noise is uncorrelated with the signal is not valid and the ENOB expression will cause a modest error

from van de Plassche (p13)

$$SNR_{corr} \cong \left(2^n - 2 + \frac{4}{\pi}\right)\sqrt{\frac{3}{2}}$$

Res (n)	SNR_{corr}	SNR
1	3.86	7.78
2	12.06	13.8
3	19.0	19.82
4	25.44	25.84
5	31.66	31.86
6	37.79	37.88
8	49.90	49.92
10	61.95	61.96

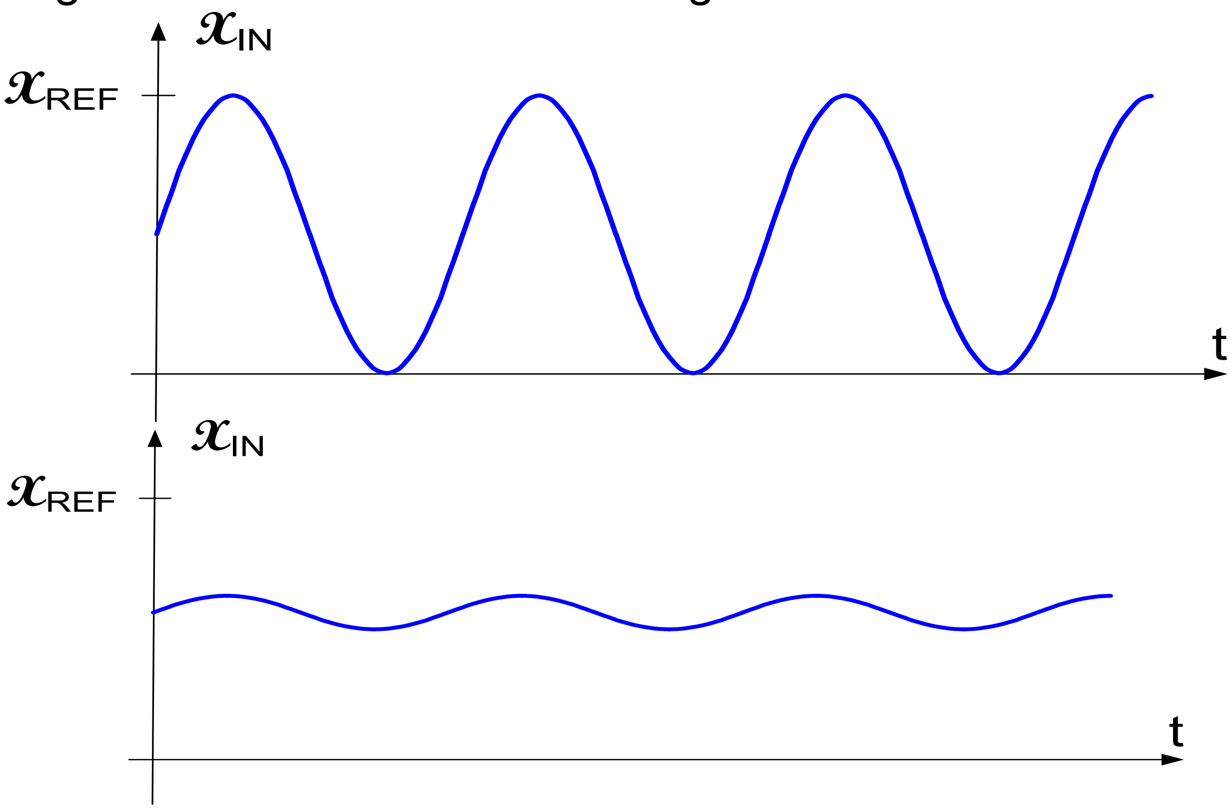
SNR = 6.02 n +1.76

Table values in dB

Almost no difference for n ≥ 3

Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude

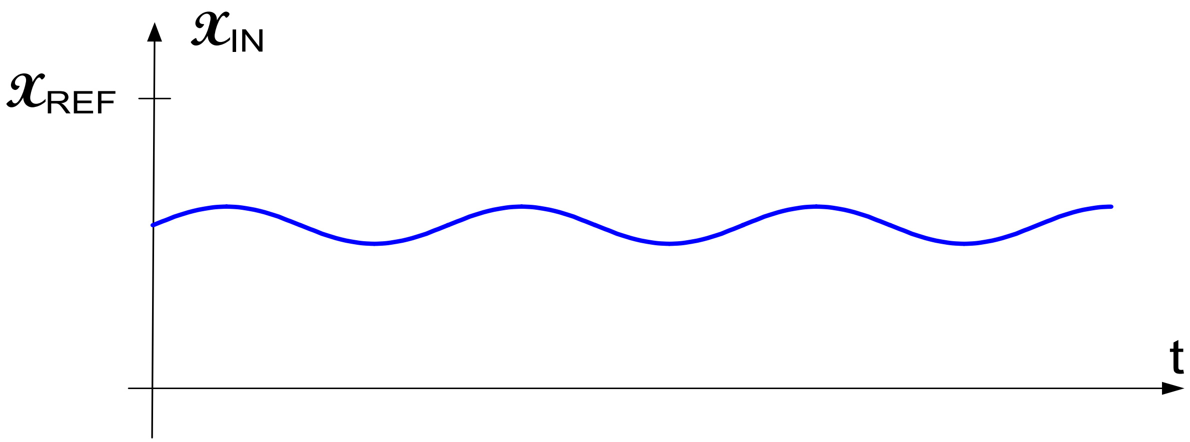


Quantization noise remains constant but signal level is reduced

The desire to use a data converter at a small fraction of full range is one of the major reasons high resolution is required

Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude



Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W output, SNR=6.02n+1.76 = 86.04dB

$$\frac{V^2}{R_L}=100W \qquad \frac{V_1^2}{R_L}=50mW \qquad V_1=\frac{V}{44.7}$$

$$20\log_{10}V_1=20\log_{10}V-20\log_{10}44.7= -33dB$$

At 50mW output, SNR reduced by 33dB to 53.04dB

$$ENOB = \frac{SNR_{dB}-1.76}{6.02} = \frac{53.04-1.76}{6.02} = 8.51$$

Note the dramatic reduction in the effective resolution of the DAC when operated at only a small fraction of full-scale.

ENOB Summary

Resolution:

$$\text{ENOB} = \frac{\log_{10} N_{ACT}}{\log_{10} 2} = \log_2 N_{ACT}$$

INL:

$$\text{ENOB} = n_R - \log_2(\upsilon) - 1$$

n_R specified res, v INL in LSB

DNL:

$$\text{ENOB} = \log_2\left(1 + \left(\frac{V_{MAX} - V_{MIN}}{\Delta_{MAX}}\right)\right)$$

V_{MAX} and V_{MIN} are max and min outputs and Δ_{MAX} is maximum absolute step (HW problem)

Quantization noise:

$$\text{ENOB} = \frac{SNR_{dB}}{6.02}$$

rel to triangle/sawtooth

$$\text{ENOB} = \frac{SNR_{dB} - 1.76}{6.02}$$

rel to sinusoid

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Absolute Accuracy

Absolute Accuracy is the difference between the actual output and the ideal or desired output of a data converter

The ideal or desired output is in reference to an absolute standard (often maintained by the National Institute of Standards and Technology (NIST) formerly Bureau of Standards) and could be volts, amps, time, weight, distance, or one of a large number of other physical quantities)

Absolute accuracy provides no tolerance to offset errors, gain errors, nonlinearity errors, quantization errors, or noise

In many applications, absolute accuracy is not of a major concern

but … scales, meters, etc. may be more concerned about Absolute accuracy than any other parameter

Relative Accuracy

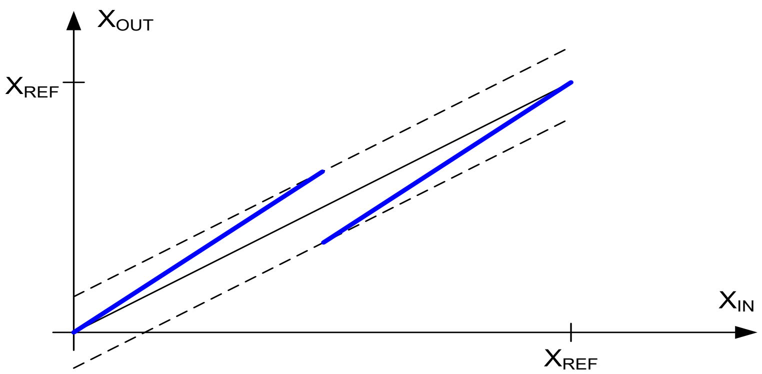
In the context of data converters, pseudo-static Relative Accuracy is the difference between the actual output and an appropriate fit-line to overall output of the data converter

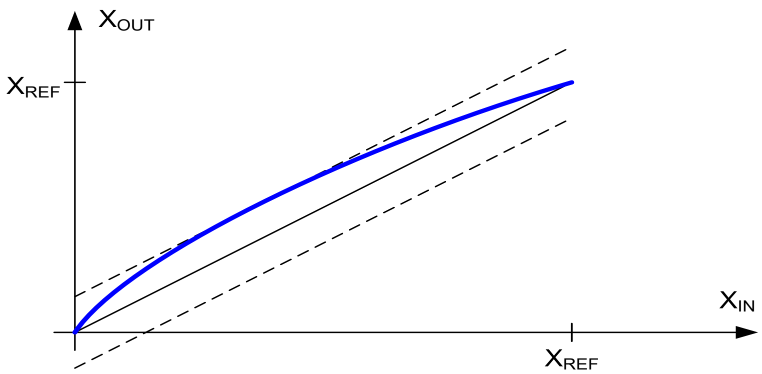
INL is often used as a measure of the relative accuracy

In many, if not most, applications, relative accuracy is of much more concern than absolute accuracy

Some architectures with good relative accuracy will have very small deviations in the outputs for closely-spaced inputs whereas others may have relatively large deviations in outputs for closely-spaced inputs

DNL provides some measure of how outputs for closely-spaced inputs compare





Stay Safe and Stay Healthy !

End of Lecture 30